B.A./B.Sc. 1st Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH1CC-I

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Find the derivative of $\tan^{-1}\tan h\frac{x}{2}$ with respect to 'x'.

(b) If $y=\frac{x}{x+1}$, find y_5 at $x=0$.

(c) Show that the curve $y=x\log_e x\ (x>0)$ is everywhere concave upwards.

(d) Find the asymptotes of $\frac{x^2}{16}-\frac{y^2}{9}=1$.

(e) Find the envelope of the straight lines $y=mx+\frac{a}{m}$, where m is the parameter and a is constant.

(f) Evaluate: $\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

(g) What is the name of the curve represented by $r^2=a^2\sin 2\theta$? Sketch it (roughly).

(h) Evaluate: $\int \tan^5 x\,dx$.

(i) Find the length of the curve $y=\log_e \sec x$ from $x=0$ to $x=\frac{\pi}{3}$.

(j) Find the rotation about the origin which will transform the equation $\sqrt{3}(x^2-y^2)-2xy=8$ into $x'\,y'=2$.

(k) Find the nature of the conic: $\frac{5}{r}=3-4\cos\theta$

(l) Determine whether the equation $y^2+z^2-2y=0$ represents a right circular cylinder or not.

(m) Obtain the differential equation of all circles each of which touches the axis of x at the origin.

(n) Find the I.F. of the ODE $y(1+xy)dx-xdy=0$.

(o) Find $f(x)$, if $f(x)+f'(x)=0$ and $f(0)=2$.

2. Answer *any four* questions from the following: 5×4=20

(a) If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x$, then prove that

(i) $(x^2-1)y_2+xy_1-m^2y=0$,

(ii) $(x^2-1)y_{n+2}+(2n+1)x\,y_{n+1}+(n^2-m^2)y_n=0$. 3+2

(b) (i) If $\sin h\,x=\tan\theta$, then show that $x=\log_e\left\{\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}$.

(ii) Find the points of inflexion on the curve $xy=a^2\log\left(\frac{y}{a}\right)$. 2+3

(c) Derive a reduction formula for $\int \sin^m x \cos^n x\,dx$, $m, n \in \mathbb{Z}^+$, $m, n \geq 2$. 5

(d) If by a rotation of rectangular axes about the origin, the expression $(ax^2 + 2hxy + by^2)$ changes to $(a'x'^2 + 2h'x'y' + b'y'^2)$, then prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$. 2+3

(e) (i) Find the whole length of the loop of the curve $3ay^2 = x(x - a)^2$.

(ii) Find the equation to the sphere with $(2,3,5)$ and $(1,2,3)$ as the end points of a diameter. Find its centre and radius. 3+2

(f) Solve: $\frac{dy}{dx} + y = y^3(\cos x - \sin x)$ 5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) State and prove Leibnitz theorem on the derivative of the product of two functions of x.

(ii) Determine the constants a and b in order that $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$.

(iii) A circle moves with its centre on the parabola $y^2 = 4ax$ and always passes through the vertex of the parabola. Show that the envelope of the circle is the curve $x^3 + y^2(x + 2a) = 0$. 4+3+3

(b) (i) If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$ and $n > 1$, show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$.

(ii) Find the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

(iii) Show that the surface area of the solid generated by the revolution about the x-axis of the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$ is 3π. 3+3+4

(c) (i) If PSP' is a focal chord of the conic $\frac{l}{r} = 1 + e\cos\theta$, prove that the angle between the tangents at P and P' is $\tan^{-1}\frac{2e\sin\alpha}{1-e^2}$, where α is the angle between the chord and the major axis.

(ii) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$.

(iii) Reducing the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to its canonical form, determine the nature of the conic for different values of a. 4+3+3

(d) (i) Solve: $\frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1 + y^2) = 0$, given that $x = 0, y = \frac{\pi}{4}$.

(ii) Solve: $y(2xy + 1)dx + x(1 + 2xy + x^2y^2)dy = 0$.

(iii) Find the general and singular solutions of the following differential equation $y = px + \sqrt{a^2p^2 + b^2}$, $p \equiv \frac{dy}{dx}$, a, b are constants. 3+3+4

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